Abstract

Why do I write yet another \texttt{R} package, when \texttt{R} itself has Bessel functions and several CRAN packages also have versions of these?

1 Introduction

\texttt{R} itself has had the function \texttt{besselI()}, \texttt{besselJ()}, \texttt{besselK()} and \texttt{besselY()}, from very early on.

However, they had shown deficiencies: First, they did only work for real (\texttt{double}) but not for \texttt{complex} arguments, even though the Bessel functions are well-defined on the whole complex plain. Second, for $x \approx 1500$ and larger, \texttt{besselI(x,nu,expon.scaled=TRUE)} jumped to zero, as I found, because of an overflow in the backward recursion (via difference equation), which I found elegantly to resolve (by re-scaling), for \texttt{R2.9.0}. However, the algorithm complexity is proportional to $|x|$, and for large $x$, a better algorithm has been desired for years. Hence, I had started experimenting with the two asymptotic expansions from \cite{Abramowitz}. 

The following \texttt{R} packages on CRAN (as of Jan.29, 2009) also provide Bessel functions:

\texttt{gsl}

\texttt{fAsianOptions}

\texttt{QRMlib} Uses many \texttt{gsl} C functions in its own code; or, rather, seems to have copy-pasted large parts of \texttt{gsl} in its own \texttt{src/} directory

2 gsl

The \texttt{R} package \texttt{gsl} by Robin Hankin provides an \texttt{R} interface on a function-by-function basis to much of the GSL, the GNU Scientific Library. You get a first overview with

\begin{verbatim}
> library(gsl)
> ?bessel_Knu
\end{verbatim}

What can I say ...

- only real 'x', not complex
- For fractional nu, the (only) interesting functions are
where the *scaled() version of each corresponds to our functions `expon.scaled=TRUE`.

- `bessel_Inu_scaled()` works for large x, comparably to our `BesselI(.)` which give warnings about accuracy loss here:
  ```r
  > x <- (1:500)*50000; b2 <- BesselI(x, pi, expo=TRUE)
  > b1 <- bessel_Inu_scaled(pi, x)
  > all.equal(b1,b2,tol=0) ## "Mean relative difference: 1.544395e-12"
  [1] "Mean relative difference: 1.849828e-12"
  > ## the accuracy is *as* limited (probably):
  > b1 <- bessel_Inu_scaled(pi, x, give=TRUE)
  > summary(b1$err)
  Min. 1st Qu. Median Mean 3rd Qu. Max.
  8.299e-08 9.580e-08 1.173e-07 1.606e-07 1.655e-07 1.856e-06
  
  where the GSL (info) manual says that `err` is an absolute error estimate, hence for relative error estimates, we look at
  ```r
  > range(b1$err/ b1$val)
  [1] 0.001040159 0.001040161
  
  So, we see that either the error estimate is too conservative, or the results only have 3 digit accuracy.

3 Session Info

> `toLatex(sessionInfo())`

- R version 3.0.2 Patched (2013-12-09 r64426), x86_64-unknown-linux-gnu
- Locale: LC_CTYPE=de_CH.UTF-8, LC_NUMERIC=C, LC_TIME=en_US.UTF-8, LC_COLLATE=C, LC_MONETARY=en_US.UTF-8, LC_MESSAGES=de_CH.UTF-8, LC_PAPER=de_CH.UTF-8, LC_NAME=C, LC_ADDRESS=C, LC_TELEPHONE=C, LC_MEASUREMENT=de_CH.UTF-8, LC_IDENTIFICATION=C
- Base packages: base, datasets, grDevices, graphics, methods, stats, utils
- Other packages: Bessel 0.5-5, Rmpfr 0.5-4, gmp 0.5-8, gsl 1.9-9
- Loaded via a namespace (and not attached): tools 3.0.2

References