Package ‘CompQuadForm’

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Title Distribution Function of Quadratic Forms in Normal Variables
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Description Computes the distribution function of quadratic forms in normal variables using Imhof’s method, Davies’s algorithm, Farebrother’s algorithm or Liu et al.’s algorithm.
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Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Davies’s method.
Usage

davies(q, lambda, h = rep(1, length(lambda)), delta = rep(0, length(lambda)), sigma = 0, lim = 10000, acc = 0.0001)

Arguments

q value point at which distribution function is to be evaluated
lambda the weights $\lambda_1, \lambda_2, ..., \lambda_n$, i.e. distinct non-zero characteristic roots of $A\Sigma$
h respective orders of multiplicity $n_j$ of the $\lambda$s
delta non-centrality parameters $\delta_j^2$ (should be positive)
sigma coefficient $\sigma$ of the standard Gaussian
lim maximum number of integration terms. Realistic values for ‘lim’ range from 1,000 if the procedure is to be called repeatedly up to 50,000 if it is to be called only occasionally
acc error bound. Suitable values for ‘acc’ range from 0.001 to 0.00005 which should be adequate for most statistical purposes.

Details

Computes $P(Q > q)$ where $Q = \sum_{j=1}^{r} \lambda_j X_j + \sigma X_0$ where $X_j$ are independent random variables having a non-central $\chi^2$ distribution with $n_j$ degrees of freedom and non-centrality parameter $\delta_j^2$ for $j = 1, ..., r$ and $X_0$ having a standard Gaussian distribution.

Value

trace vector, indicating performance of procedure, with the following components: 1: absolute value sum, 2: total number of integration terms, 3: number of integrations, 4: integration interval in main integration, 5: truncation point in initial integration, 6: standard deviation of convergence factor term, 7: number of cycles to locate integration parameters

ifault fault indicator: 0: no error, 1: requested accuracy could not be obtained, 2: round-off error possibly significant, 3: invalid parameters, 4: unable to locate integration parameters

QQ $P(Q > q)$

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, Computational Statistics and Data Analysis, Volume 54, (2010), 858-862

Examples

# Some results from Table 3, p.327, Davies (1980)

round(1 - davies(1, c(6, 3, 1), c(1, 1, 1))$qq, 4)
round(1 - davies(7, c(6, 3, 1), c(1, 1, 1))$qq, 4)
round(1 - davies(20, c(6, 3, 1), c(1, 1, 1))$qq, 4)

round(1 - davies(2, c(6, 3, 1), c(2, 2, 2))$qq, 4)
round(1 - davies(20, c(6, 3, 1), c(2, 2, 2))$qq, 4)
round(1 - davies(60, c(6, 3, 1), c(2, 2, 2))$qq, 4)

round(1 - davies(10, c(6, 3, 1), c(6, 4, 2))$qq, 4)
round(1 - davies(50, c(6, 3, 1), c(6, 4, 2))$qq, 4)
round(1 - davies(120, c(6, 3, 1), c(6, 4, 2))$qq, 4)

round(1 - davies(20, c(7, 3), c(6, 2), c(6, 2))$qq, 4)
round(1 - davies(100, c(7, 3), c(6, 2), c(6, 2))$qq, 4)
round(1 - davies(200, c(7, 3), c(6, 2), c(6, 2))$qq, 4)

round(1 - davies(10, c(7, 3), c(1, 1), c(6, 2))$qq, 4)
round(1 - davies(60, c(7, 3), c(1, 1), c(6, 2))$qq, 4)
round(1 - davies(150, c(7, 3), c(1, 1), c(6, 2))$qq, 4)

round(1 - davies(70, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)
round(1 - davies(160, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)
round(1 - davies(260, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)

round(1 - davies(-40, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)
round(1 - davies(40, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)
round(1 - davies(140, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6, 2))$qq, 4)

# You should sometimes play with the 'lim' parameter:
davies(0.00001, lambda=0.2)
imhof(0.00001, lambda=0.2)$qq
davies(0.00001, lambda=0.2, lim=20000)

farebrother

Ruben/Farebrother method

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Farebrother’s algorithm.
Usage

farebrother(q, lambda, h = rep(1, length(lambda)),
delta = rep(0, length(lambda)), maxit = 100000,
eps = 10^-10, mode = 1)

Arguments

q
value point at which distribution function is to be evaluated
lambda
the weights $\lambda_1, \lambda_2, \ldots, \lambda_n$, i.e. the distinct non-zero characteristic roots of $A\Sigma$
h
vector of the respective orders of multiplicity $m_i$ of the $\lambda$s
delta
the non-centrality parameters $\delta_i$ (should be positive)
maxit
the maximum number of term K in equation below
eps
the desired level of accuracy
mode
if 'mode' > 0 then $\beta = mode \ast \lambda_{min}$ otherwise $\beta = \beta_B = 2/(1/\lambda_{min} + 1/\lambda_{max})$

Details

Computes $P[Q>q]$ where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(m_j, \delta^2_j)$. $P[Q<q]$ is approximated by $\sum_{k=0}^{K-1} a_k P[\chi^2(m+2k) < q/\beta]$ where $m = \sum_{j=1}^{n} m_j$ and $\beta$ is an arbitrary constant (as given by argument mode).

Value

dnsty
the density of the linear form
ifault
the fault indicator. -i: one or more of the constraints $\lambda_i > 0$
$m_i > 0$ and $\delta_i^2 > 0$ is not satisfied. 1: non-fatal underflow of $a_0$. 2: one or more of the constraints $n > 0$, $q > 0$, maxit > 0 and eps > 0 is not satisfied. 3: the current estimate of the probability is greater than 2. 4: the required accuracy could not be obtained in 'maxit' iterations. 5: the value returned by the procedure does not satisfy $0 \leq RUBEN \leq 1$. 6: 'dnsty' is negative. 9: faults 4 and 5. 10: faults 4 and 6. 0: otherwise.

Qq
$P[Q > q]$

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, Computational Statistics and Data Analysis, Volume 54, (2010), 858-862

Examples

# Some results from Table 3, p.327, Davies (1980)

1 - farebrother(1, c(6, 3, 1), c(1, 1, 1), c(0, 0, 0))$q

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Imhof’s method.

Usage

imhof(q, lambda, h = rep(1, length(lambda)),
      delta = rep(0, length(lambda)),
      epsabs = 10^-6, epsrel = 10^-6, limit = 10000)

Arguments

- q: value point at which the survival function is to be evaluated
- lambda: distinct non-zero characteristic roots of $A\Sigma$
- h: respective orders of multiplicity of the $\lambda$s
- delta: non-centrality parameters (should be positive)
- epsabs: absolute accuracy requested
- epsrel: relative accuracy requested
- limit: determines the maximum number of subintervals in the partition of the given integration interval

Details

Let $X = (X_1, \ldots, X_n)'$ be a column random vector which follows a multidimensional normal law with mean vector $0$ and non-singular covariance matrix $\Sigma$. Let $\mu = (\mu_1, \ldots, \mu_n)'$ be a constant vector, and consider the quadratic form

$$Q = (x + \mu)' A (x + \mu) = \sum_{r=1}^{m} \lambda_r \chi^2_{h_r, \delta_r}.$$

The function `imhof` computes $P\{Q > q\}$.

The $\lambda_r$’s are the distinct non-zero characteristic roots of $A\Sigma$, the $h_r$’s their respective orders of multiplicity, the $\delta_r$’s are certain linear combinations of $\mu_1, \ldots, \mu_n$, and the $\chi^2_{h_r, \delta_r}$ are independent $\chi^2$-variables with $h_r$ degrees of freedom and non-centrality parameter $\delta_r$. The variable $\chi^2_{h, \delta}$ is defined here by the relation $\chi^2_{h, \delta} = (X_1 + \delta)^2 + \sum_{i=2}^{h} X_i^2$, where $X_1, \ldots, X_h$ are independent unit normal deviates.
Value

\[ P[Q > q] \]

abserr estimate of the modulus of the absolute error, which should equal or exceed abs(result)

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References


Examples

# Some results from Table 1, p.424, Imhof (1961)

# Q1 with x = 2
round(imhof(2, c(0.6, 0.3, 0.1))$Qq, 4)

# Q2 with x = 6
round(imhof(6, c(0.6, 0.3, 0.1), c(2, 2, 2))$Qq, 4)

# Q6 with x = 15
round(imhof(15, c(0.7, 0.3), c(1, 1), c(6, 2))$Qq, 4)

liu

*liu*

Liu’s method

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Liu et al.’s method.

Usage

\[
\text{liu}(q, \lambda, h = \text{rep}(1, \text{length}(\lambda)), \\
\delta = \text{rep}(0, \text{length}(\lambda)))
\]
Arguments

- **q**: value point at which the survival function is to be evaluated
- **lambda**: distinct non-zero characteristic roots of $A\Sigma$, i.e. the $\lambda_i$'s
- **h**: respective orders of multiplicity $h_i$'s of the $\lambda$'s
- **delta**: non-centrality parameters $\delta_i$'s (should be positive)

Details

New chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables.

Computes $P[Q > q]$ where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(h_j, \delta_j)$.

This method does not work as good as the Imhof’s method. Thus Imhof’s method should be recommended.

Value

- **Qq**: $P[Q > q]$

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References


Examples

```r
# Some results from Liu et al. (2009)
# Q1 from Liu et al.
round(liu(2, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)
round(liu(6, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)
round(liu(8, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)

# Q2 from Liu et al.
round(liu(1, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)
round(liu(6, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)
round(liu(15, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)

# Q3 from Liu et al.
round(liu(2, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)
round(liu(8, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)
round(liu(12, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)
```
# Q4 from Liu et al.

```
round(liu(3.5, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)), 6)
round(liu(8, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)), 6)
round(liu(13, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)), 6)
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