Package ‘boussinesq’

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License GPL (>= 2)
Title Analytic Solutions for (ground-water) Boussinesq Equation
Type Package
Depends R (>= 2.10)
Author Emanuele Cordano

Description This package is a collection of R functions implemented from published and available analytic solutions for the One-Dimensional Boussinesq Equation (ground-water). In particular, the function ‘beq.lin’ is the analytic solution of the linearized form of Boussinesq Equation between two different head-based boundary (Dirichlet) conditions; ‘beq.song’ is the non-linear power-series analytic solution of the motion of a wetting front over a dry bedrock (Song et al, 2007, see complete reference on function documentation).

Bugs/comments/questions/collaboration of any kind are warmly welcomed.

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Collate 'beq.lin.dimensionless.R' 'beq.lin.R'
         'beq.song.dimensionless.R' 'beq.song.R' 'boussinesq-package.R'
         'coefficient.song.solution.R'

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R topics documented:

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Analytic solutions for (ground-water) Boussinesq Equation

Details

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Author(s)

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beq.lin

Analytic exact solution for One-Dimensional Boussinesq Equation in a two-bounded domain with two constant-value Dirichlet Condition

Usage

beq.lin(t = 0, x = seq(from = 0, to = L, by = by),
    h1 = 1, h2 = 1, L = 100, ks = 0.01, s = 0.4,
    big = 10^7, by = L/100, p = 0.5)
Arguments

- **t**: time coordinate.
- **x**: spatial coordinate. Default is `seq(from=0L, to=L, by=by)`.
- **big**: maximum level of Fourier series considered. Default is $10^7$.
- **by**: see `seq`.
- **L**: length of the domain.
- **h1**: water surface level at $x=0$. Left Dirichlet Boundary Condition.
- **h2**: water surface level at $x=L$. Right Dirichlet Boundary Condition.
- **ks**: Hydraulic conductivity.
- **s**: drainable porosity (assumed to be constant).
- **p**: empirical coefficient to estimate hydraulic diffusivity $D = ks/\left(s \times (p \times h1 + (1 - p) \times h2)\right)$. It ranges between 0 and 1.

Value

Solutions for the indicated values of $x$ and $t$.

Author(s)

Emanuele Cordano

See Also

- `beq.lin.dimensionless`

Examples

```r
L <- 1000
x <- seq(from=0L, to=L, by=L/100)
t <- 4 # 4 days
h_sol0 <- beq.lin(x=x, t=t*24*3600, h1=2, h2=1, ks=0.01, L=L, s=0.4, big=100, p=0.0)
h_solp <- beq.lin(x=x, t=t*24*3600, h1=2, h2=1, ks=0.01, L=L, s=0.4, big=100, p=0.5)
h_sol1 <- beq.lin(x=x, t=t*24*3600, h1=2, h2=1, ks=0.01, L=L, s=0.4, big=100, p=1.0)

plot(x, h_sol0, type="l", lty=1, main=paste("Water Surface Elevation after", t, "days", sep=" "), xlab="x[m]", ylab="h[m]"
lines(x, h_solp, lty=2)
lines(x, h_sol1, lty=3)
legend("topright", lty=1:3, legend=c("p=0", "p=0.5", "p=1"))
```
beq.lin.dimensionless  Analytic exact solution for Dimensionless (i.e. diffusivity equal to 1 - unity) One Dimensional Heat Equation in a two-bounded domain with two constant-value Dirichlet Conditions

Description

Analytic exact solution for Dimensionless (i.e. diffusivity equal to 1 - unity) One Dimensional Heat Equation in a two-bounded domain with two constant-value Dirichlet Conditions

Usage

beq.lin.dimensionless(t = 0, 
  x = seq(from = 0, to = L, by = by), big = 1e+05, 
  by = L * 0.01, L = 1)

Arguments

t  time coordinate.

x  spatial coordinate. Default is seq(from=0, to=L, by=by).

big  maximum level of Fourier series considered. Default is 100000.

by  see seq

L  length of the domain. It is used if x is not specified.

Value

Solutions for the specified values of x and t

Author(s)

Emanuele Cordano

References


See Also

beq.lin
Song et al.’s analytic solution to Boussinesq equation in a 1D semi-infinite domain with a Dirichlet boundary condition

**Description**

Song et al.’s analytic solution to Boussinesq equation in a 1D semi-infinite domain with a Dirichlet boundary condition

**Usage**

beq.song(t = 0.5, x = 1, s = 0.4, h1 = 1, ks = 0.01, nmax = 4, alpha = 1)

**Arguments**

- **t**  
  time coordinate.
- **x**  
  spatial coordinate. Default is seq(from=0, to=L, by=by).
- **h1**  
  water surface level or boundary condition coefficient at x=0. Left Dirichlet Boundary Condition.
- **ks**  
  Hydraulic conductivity
- **s**  
  drainable porosity (assumed to be constant)
- **nmax**  
  order of power series considered for the analytic solution. Default is 4.
- **alpha**  
  $\alpha$ exponent see Song et al., 2007

**Value**

The water surface elevation vs time and space obtained by the analytic solution of Boussinesq Equation

**Note**

For major details, see Song at al., 2007

**Author(s)**

Emanuele Cordano

**References**

Song, Zhi-yao; Li, Ling; David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation", Applied Mathematics and Mechanics, http://www.springerlink.com/content/w0u8667772712801/, http://dx.doi.org/10.1007/s10483-007-6012-x

**See Also**

beq.song.dimensionless
**Examples**

```r
L <- 1000
x <- seq(from=0, to=L, by=L/100)
t <- c(4,5,20) # days

h_sol1 <- beq.song(t[1]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)
h_sol2 <- beq.song(t[2]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)
h_sol3 <- beq.song(t[3]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)

plot(x,h_sol1,type="l",lty=1,main="Water Surface Elevation (Song at's solution) ",xlab="x[m]",ylab="h[m]")
lines(x,h_sol2,lty=2)
lines(x,h_sol3,lty=3)
legend("topright",lty=1:3,legend=paste("t","days",sep= " "))
```

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**beq.song.dimensionless**

*Dimensionless solution for one-dimensional derived equation from scaling Boussinesq Equation (Song et al, 2007)*

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**Description**

Dimensionless solution for one-dimensional derived equation from scaling Boussinesq Equation (Song et al, 2007)

**Usage**

```r
beq.song.dimensionless(xi, xi0, a)
```

**Arguments**

- `xi`: dimensionless coordinate (see Note)
- `xi0`: displacement of wetting front expressed as dimensionless coordinate (see Note)
- `a`: vector of coefficient returned by `coefficient.song.ssolution`

**Value**

the dimensionless solution, i.e. the variable `H`

**Note**

The expression for the dimensionless coordinate (Song et al., 2007) is

$$\xi = x \left( \frac{2 s}{\eta \alpha + 1} \right)^{1/2}$$

and the solution for the dimensionless equation derived by Boussinesq Equation is:

$$H = \sum_{n=0}^{\infty} a_n \left(1 - \frac{\xi}{\xi_0}\right)^n$$

for $$\xi < \xi_0$$, otherwise is 0.

**Author(s)**

Emanuele Cordano
References

Song, Zhi-yao; Li, Ling; David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation", Applied Mathematics and Mechanics, http://www.springerlink.com/content/w0u8667772712801/, http://dx.doi.org/10.1007/s10488-007-0612-x

See Also

beq.song

Description

Algorithm for resolution of the series coefficient $a_n$ for the dimensionless formula for $H$ in beq.song.dimensionless

Usage

coefficient.song.solution(n = 4, lambda = 0)

Arguments

- **n**
  - approximation order
- **lambda**
  - dimensionless parameter related to $\alpha$ see Song et al., 2007

Value

the $a_n$ series coefficient

Note

For major details, see Song et al., 2007

Author(s)

Emanuele Cordano

References

Song, Zhi-yao; Li, Ling; David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation", Applied Mathematics and Mechanics, http://www.springerlink.com/content/w0u8667772712801/, http://dx.doi.org/10.1007/s10488-007-0612-x
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