Package ‘conics’

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Type Package

Title Plot Conics

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Description plot conics (ellipses, hyperbolas, parabolas)

License GPL (>= 2)

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conicAsymptotes

Asymptotes of a conic

Description
Find the slopes of the asymptotic directions of a conic.

Usage
conicAsymptotes(x)

Arguments
x a 6-length vector or a symmetric 3x3 matrix

Details
The conicAsymptotes function calculates the slopes of the asymptotic directions of a conic specified by its coefficients or by its symmetric matrix.
If the equation of the conic is
\[ v_1 x_1^2 + v_2 x_1 x_2 + v_3 x_2^2 + v_4 x_1 + v_5 x_2 + v_6 = 0 \]
the slopes of the asymptotes are the roots of the equation at infinity of the conic:
\[ v_1 + v_2 t + v_3 t^2 = 0 \]
where \( t = x_2/x_1 \).

Value
A vector containing the slopes: two values in the case of a hyperbola or of intersecting lines, one value in the case of a parabola or of parallel lines. In the case of an ellipse (which has no points at infinity), the function returns an empty vector.

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See Also
conicAxes, conicCenter, conicMatrix, conicPlot
conicAxes

Examples

# Hyperbola
# Equation: 2*x_1^2 + 2*x_1*x_2 - 2*x_2^2 - 2/zero.noslash*x_1 + 2/zero.noslash*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,-2,-2/zero.noslash,2/zero.noslash,1/zero.noslash)
conicAsymptotes(v)

# Ellipse
# Equation: 2*x_1^2 + 2*x_1*x_2 + 2*x_2^2 - 2/zero.noslash*x_1 - 28*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,2,-2/zero.noslash,-28,1/zero.noslash)
# Should return an empty vector (an ellipse has no asymptotes!):
conicAsymptotes(v)

conicAxes

Axes of a conic

Description

Find the symmetry axes of a conic.

Usage

conicAxes(x)

Arguments

x

a 6-length vector or a symmetric 3x3 matrix

Details

The conicAxes function calculates the coordinates of the symmetry axes of a conic specified by its coefficients or by its symmetric matrix.

The direction vectors of the axes are the eigenvectors of the top-left 2x2 submatrix of the matrix representing the conic.

Value

A 2x2 matrix whose columns are the direction vectors of the axes. In order to find the coordinates of the center, see the function conicCenter.

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See Also

conicAsymptotes, conicCenter, conicMatrix, conicPlot
Examples

# Ellipse
# Equation: 2*x_1^2 + 2*x_1*x_2 + 2*x_2^2 - 2/zero.noslash*x_1 - 28*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,-20,-28,10)
conicAxes(v)

# Hyperbola
# Equation: 2*x_1^2 + 2*x_1*x_2 - 2*x_2^2 - 2/zero.noslash*x_1 + 2/zero.noslash*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,-2,-20,20,10)
conicAxes(v)

conicCenter  Center of a conic

Description
Find the center of a conic.

Usage
conicCenter(x)

Arguments
x  a 6-length vector or a symmetric 3x3 matrix

Details
The conicCenter function calculates the coordinates of the center of a conic specified by its coefficients or by its symmetric matrix.

Value
A two-elements vector containing the coordinates of the center. If the conic has no center the function raises an error.

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See Also
conicAsymptotes, conicAxes, conicMatrix, conicPlot
Conic matrix

Examples

# Ellipse
# Equation: 2*x_1^2 + 2*x_1*x_2 + 2*x_2^2 - 2/zero.noslash*x_1 - 28*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,2,-2/zero.noslash,-28,1/zero.noslash)
conicCenter(v)

# Hyperbola
# Equation: 2*x_1^2 + 2*x_1*x_2 - 2*x_2^2 - 2/zero.noslash*x_1 + 2/zero.noslash*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,-2,-2/zero.noslash,2/zero.noslash,1/zero.noslash)
conicCenter(v)

Conic matrix

Matrix representing a conic

Description

Build a symmetric matrix representing a quadratic polynomial in two variables.

Usage

conicMatrix(v)

Arguments

v (vector) a 6-length vector containing the coefficients of a quadratic polynomial.

Details

The \( v \) argument is a 6-length vector containing the coefficients of a quadratic polynomial of the form:

\[
P(x_1, x_2) = v_1 x_1^2 + v_2 x_1 x_2 + v_3 x_2^2 + v_4 x_1 + v_5 x_2 + v_6
\]

The associated quadratic form is:

\[
Q(x_1, x_2, x_3) = v_1 x_1^2 + v_2 x_1 x_2 + v_3 x_2^2 + v_4 x_1 x_3 + v_5 x_2 x_3 + v_6 x_3^2
\]

Value

Return the symmetric 3x3 matrix representing the associated quadratic form.

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See Also

conicAsymptotes, conicAxes, conicCenter, conicPlot
**Examples**

```r
# Equation: 2*x_1^2 + 2*x_1*x_2 + 2*x_2^2 - 2/zero.noslash*x_1 - 28*x_2 + 1/zero.noslash = /zero.noslash
v <- c(2,2,2,-2/zero.noslash,-28,1/zero.noslash)
conicMatrix(v)
```

---

**Description**

Plot a conic (ellipse, hyperbola, or parabola) specified by a quadratic polynomial or by a symmetric 3x3 matrix.

**Usage**

```r
conicPlot(x, type=l, npoints=1/zero.noslash/zero.noslash, sym.axes=FALSE, center=FALSE, asymptotes=FALSE, add=FALSE, xlim=NULL, ylim=NULL, ax.lty=1, ax.col=palette()[1], as.lty=1, as.col=palette()[1], ...)
```

**Arguments**

- `x`: a 6-length vector or a symmetric 3x3 matrix
- `type`: (character) the type of plot to draw (same meaning as with the `plot` function)
- `npoints`: (numeric) number of points to draw
- `sym.axes`: (logical) if TRUE, display the axes of the conic
- `center`: (logical) if TRUE, display the center of the conic (if any)
- `asymptotes`: (logical) if TRUE, display the asymptotes (hyperbolas)
- `add`: (logical) if TRUE, plot over the current graphical device
- `xlim`: (vector) interval for the x-coordinate
- `ylim`: (vector) interval for the y-coordinate
- `ax.lty`: (character or numeric) line type of the axes
- `ax.col`: (character or numeric) color of the axes
- `as.lty`: (character or numeric) line type of the asymptotes
- `as.col`: (character or numeric) color of the asymptotes
- `...`: other parameters passed to the `plot` function

**Details**

The `conicPlot` function identifies the type of the conic and plots it in the current graphical device. The conic is specified either by a 6-length vector representing the coefficients of the quadratic polynomial, or by the symmetric matrix representing the associated quadratic form. See the function `conicMatrix` to build this matrix given the coefficients of the polynomial.

It is usually a good idea to set explicitly the aspect ratio to 1 (as an additional argument `asp=1` in the `conicPlot` function) in order to avoid distortions between the units of the x-axis and the y-axis. See examples below.
conicPlot

Value

The return value is invisible, i.e. it is not printed on the console by default but can be stored in a variable. It is a list of relevant computed values corresponding to various elements of the conic. The following elements can be found in the return list, depending on the kind of the conic:

- **kind** the kind of the conic: "ellipse", "hyperbola", "parabola", or "lines".
- **axes** the symmetry axes. See also the function `conicAxes`.
- **center** the center of the conic. See also the function `conicCenter`.
- **asymptotes** the slopes of the asymptotes. See also the function `conicAsymptotes`.
- **vertices** the vertices of the conic.
- **foci** the focal points of the conic.
- **eccentricity** the eccentricity of the conic.
- **intercepts** the intercepts in the case of parallel lines.
- **points** the coordinates of the points used to plot the conic.

The points component is returned only if the `type` option is equal to n and if the conic is non-degenerate. In that case, nothing is drawn.

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See Also

conicAsymptotes, conicAxes, conicCenter, conicMatrix

Examples

```r
# Ellipse
# -------
# Equation: 2*x_1^2 + 2*x_1*x_2 + 2*x_2^2 - 20*x_1 - 28*x_2 + 10 = 0
type <- c(2,2,2,-2,28,1)
conicPlot(type)
type[6] <- 20
conicPlot(type, type=p, col="red", add=TRUE)

# Symmetric matrix
m <- rbind( c(5, -3, -21),
            c(-3, 5, -19),
            c(-21, -19, 93) )
conicPlot(m)

# Hyperbola
# -------
# Equation: 2*x_1^2 + 2*x_1*x_2 - 2*x_2^2 - 20*x_1 + 20*x_2 + 10 = 0
type <- c(2,2,-2,20,20,10)
conicPlot(type, center=TRUE, sym.axes=TRUE, asp=1)
conicPlot(type, asymptote=TRUE, as.col="grey30", as.lty=2,
          sym.axes=TRUE, ax.col="red", ax.lty=6, col="blue", asp=1)
```
# Parabola
# --------
# Equation: \(4x_1^2 + 4x_1x_2 + x_2^2 + 2/\text{zero.noslash}x_1 + 2/\text{zero.noslash}x_2 + 2/\text{zero.noslash} = \text{zero.noslash}\)
\(v \leftarrow c(4,4,1,20,20,20)\)
conicPlot(v, sym.axes=TRUE, ax.lty=2, asp=1)

# Degenerate conics
# -----------------
# Intersecting lines
# Equation: \(x_1^2 - 2x_1x_2 - 8x_2^2 - 2x_1 + 14x_2 - 3 = \text{zero.noslash}\)
\(v \leftarrow c(1,-2,-8,-2,14,-3)\)
conicPlot(v)

# Parallel lines
# Equation: \(x_1^2 - 2x_1x_2 + x_2^2 + 4x_1 - 4x_2 + 3 = \text{zero.noslash}\)
\(v \leftarrow c(1,-2,1,4,-4,3)\)
conicPlot(v)

# Coincident lines
# Equation: \(4x_1^2 + 12x_1x_2 + 9x_2^2 - 4x_1 - 6x_2 + 1 = \text{zero.noslash}\)
\(v \leftarrow c(4,12,9,-4,-6,1)\)
conicPlot(v)

# Return value
# ------------
\(v \leftarrow c(2,2,2,-20,-28,10)\)
cp <- conicPlot(v)
cp$kind
cp$vertices
cp$center
cp$axes
cp <- conicPlot(v,type=n)
cp$points

---

**conics** ~**Conics plotting**~

**Description**

* Package: conics  
* Type: Package  
* Version: 0.3  
* Date: 2013-12-10  
* License: GPL (>= 2)

**Details**

The conics package provides simple functions to plot conics. A conic is a plane algebraic curve of degree 2: it is the set of zeroes of a polynomial of degree 2 in 2 variables, that is to say the set of points \((x_1, x_2)\) satisfying an equation of the form
Non-degenerate conics include the ellipses, the hyperbolas and the parabolas. Degenerate conics are pairs of lines.

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References

For more information about the algebraic background of conics and their matrix representation, see the vignette accompanying this package. To display the vignette, type the following instruction in the R console:

> vignette("conics")

See Also

The following functions are available: conicAsymptotes, conicAxes, conicCenter, conicMatrix, conicPlot
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