Package ‘orthogonalsplinebasis’

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Description Represents the basis functions for B-splines in a simple matrix formulation that facilitates, taking integrals, derivatives, and making orthogonal the basis functions.
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A Matrix Representation for Spline Basis Functions

Description
This package provides functions for manipulation of spline basis functions. A matrix representation for the basis functions is at the center of the functions. The matrix representation simplifies the process of orthogonalization as well as differentiation and integration.

Author(s)
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Methods

**object = "SplineBasis", x = "numeric"** Evaluates a SplineBasis object for the spline basis curves at points x. See `SplineBasis`

Usage
```
expand.knots(interior, order = 4)
```

Arguments
- **interior**: The knots including all interior and endpoint knots
- **order**: the order of the splines that the knots are to be used with. Defaults to 4, being cubic splines
Value

A vector of knots with the order specified as an attribute

Author(s)

Andrew Redd

See Also

SplineBasis

Examples

(knots<-expand.knots(1:10))
plot(0Basis(knots))

fitLS Fitting splines with penalized least squares.

Description

Estimates the control vector for a spline fit by penalized least squares. The penalty being the penalty parameter times the functional inner product of the second derivative of the spline curve.

Usage

fitLS(object, x, y, penalty = 0)

Arguments

object The SplineBasis object ot be used to make the fit
x predictor variable.
y response variable.
penalty The penalty multiplier.

Details

For numeric vector y, and x, and a set of basis functions, represented in object, defined on the knots \((k_0, \ldots, k_m)\). The likelihood is defined by

\[
\sum_{i=1}^{n}(y_i - b(x_i)\mu) + \int_{k_0}^{k_m} \mu^T b''(t)T b''(t)\mu dt
\]

The function estimates \(\mu\).
Function for computing the Gram matrix of a spline basis.

**Example**

```r
knots <- c(0, 0, 0, 5, 5, 5)
base <- SplineBasis(knots)
x <- seq(0, 5, by = .5)
y <- exp(x) + rnorm(length(x), sd = 5)
fitLS(base, x, y)
```

**Description**

Function for computing the Gram matrix of a spline basis.

**Usage**

```r
GramMatrix(object)
```

**Arguments**

- `object` a `SplineBasis` object

**Details**

Compute the Gram Matrix. If `object` denotes the basis functions \( b(t) = \{b_1(t), \ldots, b_J(t)\} \) then the Gram Matrix is,

\[
G = \int b^T(t) b(t) \, dt
\]

**Value**

a matrix as defined above.
Generating a Hankel Matrix

Description

Functions to generate a Hankel matrix.

Usage

\[ \text{Hankel}(x, \text{nrow} = \text{length}(x)/2, \text{ncol} = \text{length}(x)/2) \]

Arguments

- **x**: numeric vector to specify the entries of the matrix. Should have an even number of entries.
- **nrow**: integer, must be at most length(x)
- **ncol**: integer, must be at most length(x)

Details

Computes a Hankel matrix. If we denote the vector \( x = (x_1, \ldots, x_n) \) the Hankel matrix is defined and formed as

\[
H = \begin{pmatrix}
x_1 & x_2 & x_3 & \cdots & x_{1/2} \\
x_2 & x_3 & \vdots & \ddots \\
x_3 & \vdots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots \\
x_{1/2} & \cdots & \cdots & \cdots & x_n
\end{pmatrix}.
\]

Value

A matrix as defined above.

Author(s)

Andrew Redd <aredd at stat.tamu.edu>

Examples

\[ \text{Hankel}(1:6) \]
Methods for Function `integrate`

Description

Methods for function `integrate`. `integrate` integrates generic objects for which an integral is defined.

Methods

- `object = "SplineBasis"` Returns a new SplineBasis object for the integral of the basis functions. See `SplineBasis`

---

Matrix Power

**Description**

Performs the matrix power operation.

**Usage**

`MatrixPower(A, n)`

**Arguments**

- `A` A square matrix.
- `n` An integer telling the exponent.

**Details**

Only well defined for integers the matrix power operation is a convenience function to multiply a matrix, `A`, with itself `n` times.

**Value**

A matrix of the same dimension as `A`.

**Author(s)**

Andrew Redd <aredd at stat.tamu.edu>
Examples

```r
A <- rbind(0, cbind(diag(1:5), 0))  # a nilpotent matrix
A
MatrixPower(A, 3)
MatrixPower(A, 5)
MatrixPower(A, 6)  # Gets to a zero matrix
```

Description

A generic function for orthogonalizing an object and returning the orthogonal object.

Methods

- `object = "SplineBasis"` orthogonalizes the spline basis functions. See `SplineBasis`

OrthogonalizeBasis  Orthogonalize a Spline Basis

Description

Specific function for orthogonalizing the functions in a SplineBasis object.

Usage

```r
OrthogonalizeBasis(object, ...)
```

Arguments

- `object` A SplineBasis object
- `...` ignored

Value

An `OrthogonalSplineBasis` object.

Author(s)

Andrew Redd <aredd at stat.tamu.edu>

See Also

`OrthogonalSplineBasis, SplineBasis, orthogonalize`
OuterProdSecondDerivative

*Outer Product of Second Derivatives of Spline Bases*

**Description**

Provides the functional outer product of second derivatives of a set of basis functions in a SplineBasis object. It a convenient form for forming a penalty on curve smoothness when fitting a spline curve.

**Usage**

`OuterProdSecondDerivative(basis)`

**Arguments**

- `basis` A SplineBasis object

**Value**

A square matrix of order `nrow(basis)`.

**Author(s)**

Andrew Redd <aredd at stat.tamu.edu>

**See Also**

`splinebasis`, `fitLS`

---

**SplineBasis**

*Creating and SplineBasis Objects.*

**Description**

The function to create SplineBasis and OrthogonalSplineBasis Objects

**Usage**

```r
SplineBasis(knots, order=4, keep.duplicates=FALSE)  
OrthogonalSplineBasis(knots, ...)  
0Basis(...)```
Arguments

- **knots**: The full set of knots used to define the basis functions.
- **order**: Order of the spline fit. (degree = order - 1)
- **keep.duplicates**: Should duplicate interior knots that could cause computation problem be kept or removed. Defaults to false, which removes duplicate knots with a warning if duplicate interior knots are found.
- **...**: Other arguments either ignored or passed onto other functions.

Details

SplineBasis produces an object representing the basis functions used in spline fitting. Provides a compact easily evaluated representation of the functions. Produces a class of object SplineBasis. OrthogonalSplineBasis is a shortcut to obtain a set of orthogonalized basis functions from the knots. OBasis is an alias for OrthogonalSplineBasis. Both provide an object of class OrthogonalSplineBasis. The class OrthogonalSplineBasis inherits directly from SplineBasis meaning all functions that apply to SplineBasis functions also apply to the orthogonalized version.

Value

Object of class SplineBasis or OrthogonalSplineBasis

References


See Also

SplineBasis, spline, orthogonalsplinebasis-package

Examples

```r
knots <- c(0, 0, 0:10, 10, 10)
plot(SplineBasis(knots))
obase <- OBasis(knots)
plot(obase)
dim(obase)[2] # number of functions
evaluate(obase, 1:10-.5)
```

Description

Contains the matrix representation for spline basis functions. The OrthogonalSplineBasis class has the basis functions orthogonalized.
SplineBasis-class

Objects from the Class

Objects can be created by calls of the form \texttt{SplineBasis(knots, order)} or to generate orthogonal spline basis functions directly \texttt{OrthogonalSplineBasis(knots, order)} or the short version \texttt{OBasis(knots,order)}.

Slots

- \texttt{transformation}: Object of class "matrix" Only applicable on OrthogonalSplineBasis Class, shows the transformation matrix use to get from regular basis functions to orthogonal basis functions.
- \texttt{knots}: Object of class "numeric"
- \texttt{order}: Object of class "integer"
- \texttt{Matrices}: Object of class "array"

Methods

- \texttt{deriv signature(expr = "SplineBasis"): Computes the derivative of the basis functions. Returns an object of class SplineBasis.}
- \texttt{dim signature(x = "SplineBasis"): gives the dim as the order and number of basis functions. Returns numeric of length 2.}
- \texttt{evaluate signature(object = "SplineBasis", x = "numeric"): Evaluates the basis functions and the points provided in x. Returns a matrix with length(x) rows and dim(object)[2] columns.}
- \texttt{integrate signature(object = "SplineBasis"): computes the integral of the basis functions defined by \[ \int_{k_0}^x b(t)dt \] where \( k_0 \) is the first knot. Returns an object of class SplineBasis.}
- \texttt{orthogonalize signature(object = "SplineBasis"): Takes in a SplinesBasis object, computes the orthogonalization transformation and returns an object of class OrthogonalSplineBasis.}
- \texttt{plot signature(x = "SplineBasis", y = "missing"): Takes an object of class SplineBasis and plots the basis functions for the domain defined by the knots in object.}
- \texttt{plot signature(x = "SplineBasis", y = "vector"): Interprets y as a vector of coefficients and plots the resulting curve.}
- \texttt{plot signature(x = "SplineBasis", y = "matrix"): Interprets y as a matrix of coefficients and plots the resulting curves.}

Author(s)

Andrew Redd <aredd at stat.tamu.edu>

References


See Also

SplineBasis
**SplineBasis-class**

**Examples**

```r
showClass("SplineBasis")

knots <- c(0, 0, 0, 5, 5, 5)
(base <- SplineBasis(knots))
(obase <- OBasis(knots))
plot(base)
plot(obase)
```
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