Package ‘polynom’

March 22, 2019

Version 1.4-0
Title A Collection of Functions to Implement a Class for Univariate Polynomial Manipulations
Description A collection of functions to implement a class for univariate polynomial manipulations.
Imports stats, graphics
License GPL-2
NeedsCompilation no
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Suggests knitr, rmarkdown
VignetteBuilder knitr
Repository CRAN
Date/Publication 2019-03-22 09:50:06 UTC

R topics documented:

as.function.polynomial ............................................. 2
change.origin .......................................................... 3
deriv.polynomial ....................................................... 4
gcd .............................................................. 5
integral.polynomial ................................................... 5
lines.polynomial ....................................................... 6
Math.polynomial ....................................................... 7
monic .............................................................. 8
Ops.polynomial ......................................................... 9
plot.polynomial ....................................................... 9
points.polynomial .................................................... 10
poly.calc .......................................................... 11
poly.orth .......................................................... 12
as.function.polynomial

Coerce a Polynomial to a Function

Description

Takes a polynomial argument and constructs an R function to evaluate it at arbitrary points.

Usage

## S3 method for class 'polynomial'
as.function(x, ...)

Arguments

x      An object of class "polynomial".
...

further arguments to be passed to or from methods.

Details

This is a method for the generic function as.function.

The polynomial is evaluated within the function using the Horner scheme.

Note that you can use the model-oriented predict method for polynomials for purpose of evaluation (without explicit coercion to a function), see the example below.

Value

A function to evaluate the polynomial p.

See Also

as.function, predict.polynomial
Examples

```r
pr <- (poly.calc(-1:1) - 2 * polynomial(c(1, 2, 1)))^2
pr
## 4 + 20*x + 33*x^2 + 16*x^3 - 6*x^4 - 4*x^5 + x^6
prf <- as.function(pr)
prf
## function (x)
## 4 + x * (20 + x * (33 + x * (16 + x * (-6 + x * (-4 + x * (1))))))
## <environment: 0x4024f0>
prf(-3:3)
## 1024 64 0 4 64 144 64
predict(prf, -3:3)
## 1024 64 0 4 64 144 64
```

change.origin

Change Origin for a Polynomial

Description

Calculate the coefficients of a polynomial relative to a new origin on the x axis.

Usage

`change.origin(p, o)`

Arguments

- `p` an object of class "polynomial".
- `o` a numeric scalar representing the new origin on the original scale.

Details

Let `P(x) = \sum p_i x^i` be a given polynomial and consider writing `P(x) = \sum q_j (x - o)^j`. This function calculates the coefficients `q_j` and returns the result as a polynomial.

Value

A polynomial with coefficients relative to the re-located x axis.

Examples

```r
pr <- poly.calc(1:5)
pr
## -120 + 274*x - 225*x^2 + 85*x^3 - 15*x^4 + x^5
change.origin(pr, 3)
## 4*x - 5*x^3 + x^5
```
**deriv.polynomial**

*Differentiate a Polynomial*

**Description**

Calculates the derivative of a univariate polynomial.

**Usage**

```r
## S3 method for class 'polynomial'
deriv(expr, ...)
```

**Arguments**

- `expr`: an object of class "polynomial".
- `...`: further arguments to be passed to or from methods.

**Details**

This is a method for the generic function `deriv`.

**Value**

Derivative of the polynomial.

**See Also**

`integral.polynomial`, `deriv`.

**Examples**

```r
pr <- poly.calc(1:5)
pr
## -120 + 274*x - 225*x^2 + 85*x^3 - 15*x^4 + x^5
deriv(pr)
## 274 - 450*x + 255*x^2 - 60*x^3 + 5*x^4
```
gcd

GCD and LCM for Polynomials

Description

Compute the greatest common divisor (GCD) and least common multiple (LCM) of a collection of polynomials and polylists.

Usage

\[
\begin{align*}
\texttt{gcd}(\ldots) & \quad \text{method for class 'polylist'}
\texttt{GCD}(\ldots) \\
\texttt{gcd}(\ldots) & \quad \text{method for class 'polynomial'}
\texttt{GCD}(\ldots) \\
\texttt{lcm}(\ldots) & \quad \text{method for class 'polylist'}
\texttt{LCM}(\ldots) \\
\texttt{lcm}(\ldots) & \quad \text{method for class 'polynomial'}
\texttt{LCM}(\ldots)
\end{align*}
\]

Arguments

\[\ldots\] a list of objects of class \texttt{polynomial} or \texttt{polylist}.

Examples

\[
\begin{align*}
\texttt{pl} & \leftarrow \texttt{polylist}(& \texttt{poly.from.roots}(-1), \\
& \texttt{poly.from.roots}(c(-1, -1)), \\
& \texttt{poly.from.roots}(1))
\texttt{GCD}(\texttt{pl}) \\
\texttt{GCD}(\texttt{pl}[-3]) \\
\texttt{LCM}(\texttt{pl}) \\
\texttt{LCM}(\texttt{pl}, \texttt{pl}, \texttt{pl}[2])
\end{align*}
\]

integral.polynomial

Integrate a Polynomial

Description

Find the integral of a univariate polynomial.

Usage

\[
\begin{align*}
\texttt{integral}(\texttt{expr}, \texttt{limits = NULL}, \ldots)
\end{align*}
\]
Arguments

expr an object of class "polynomial".

limits numeric vector of length 2 giving the integration limits.

... further arguments to be passed to or from methods.

Value

If limits is not given, the integral of p from 0 to ‘x’. Otherwise, the integral with the given integration limits.

See Also

deriv.polynomial

Examples

p <- poly.calc(1:5)
p
## -120 + 274*x - 225*x^2 + 85*x^3 - 15*x^4 + x^5
deriv(p)
## 274 - 450*x + 255*x^2 - 60*x^3 + 5*x^4
integral(deriv(p)) - 120
## -120 + 274*x - 225*x^2 + 85*x^3 - 15*x^4 + x^5

lines.polynomial

Description

Add a polynomial to an existing plot usually as a line plot.

Usage

## S3 method for class 'polynomial'
lines(x, len = 1000, xlim = NULL, ylim = NULL, ...)

Arguments

x an object of class "polynomial".

len size of vector at which evaluations are to be made.

xlim, ylim the range of x and y values with sensible defaults.

... additional arguments as for the lines generic.
Math.polynomial

Details

This is a method for the generic function lines.

Lines representing the given polynomial are added to an existing plot. Values outside the current plot region are not shown.

See Also

lines, points, points.polynomial, plot, plot.polynomial.

Examples

plot (poly.calc(-1:5))
lines (poly.calc( 2:4), lty = 2)
points(poly.calc(-2:6), pch = 4)

Math.polynomial  Math Group Methods for Polynomials

Description

Group method for functions in the Math group.

Usage

## S3 method for class 'polynomial'
Math(x, ...)

Arguments

x an object of class "polynomial".
... further arguments to be passed to or from methods, such as digits.

Details

Most math group functions are disallowed with polynomial arguments. The only exceptions are ceiling, floor, round, trunc, and signif which may be used to transform the coefficients accordingly.

Value

A polynomial with transformed coefficients.

See Also

Ops.polynomial, Summary.polynomial.
monic

Monic Polynomials

Description

Convert a polynomial to monic form by dividing by the leading coefficient.

Usage

monic(p)

Arguments

p

A polynomial. A warning is issued if the polynomial is identically zero.

Details

Similar in effect to \( p / as.numeric(p[length(p)]) \) but with some safeguards against leading zero coefficients.

Value

A polynomial proportional to \( p \) with leading coefficient 1.

See Also

change.origin

Examples

op <- options(digits=18)
p <- poly.from.values(1:4, (2:5)^2)
## 1 + 2.00000000000001*x + x^2
p <- round(p)
## 1 + 2*x + x^2
options(op)
Ops.polynomial

Arithmetic Ops Group Methods for Polynomials

Description

Allows arithmetic operators to be used for polynomial calculations, such as addition, multiplication, division, etc.

Usage

```r
## S3 method for class 'polynomial'
Ops(e1, e2)
```

Arguments

- `e1`: an object of class "polynomial".
- `e2`: an object of class "polynomial".

Value

A polynomial got by performing the operation on the two arguments.

See Also

`Math.polynomial`, `Summary.polynomial`

Examples

```r
p <- polynomial(c(1, 2, 1))
## 1 + 2*x + x^2
r <- poly.calc(-1 : 1)
## -1*x + x^3
(r - 2 * p)^2
## 4 + 20*x + 33*x^2 + 16*x^3 - 6*x^4 - 4*x^5 + x^6
```

plot.polynomial

Plot Method for Polynomials

Description

Plots polynomials, optionally allowing the “interesting” region to be automatically determined.

Usage

```r
## S3 method for class 'polynomial'
plot(x, xlim = 0:1, ylim = range(px), type = "l",
     len = 1000, ..., xlab = "x", ylab = "P(x)")
```
Arguments

- **x**: an object of class "polynomial".
- **xlim**: the range to be encompassed by the x axis.
- **ylim**: the range to be encompassed by the y axis.
- **type**: as for `plot()`.
- **len**: number of x points drawn.
- **...**: additional arguments as for `plot`.
- **xlab**, **ylab**: graphical parameters.

Details

This is a method for the generic function `plot`.

A plot of the polynomial is produced on the currently active device. Unless otherwise specified, the domain is chosen to enclose the real parts of all zeros, stationary points and zero itself.

See Also

`plot`, `lines`, `points`, `lines.polynomial`, `points.polynomial`.

Examples

```r
plot(p <- poly.calc(-1:5))
```

---

Description

Add a polynomial to an existing plot usually as a point plot.

Usage

```r
## S3 method for class 'polynomial'
points(x, length = 100, ...)
```

Arguments

- **x**: an object of class "polynomial".
- **length**: size of x vector at which evaluations are to be made.
- **...**: additional arguments as for the points generic.

Details

This is a method for the generic function `points`.

Points representing the given polynomial are added to an existing plot. Values outside the current plot region are not shown.
poly.calc

See Also

plot, lines, points, plot.polynomial, lines.polynomial.

Examples

plot(poly.calc(-1:5))
lines(poly.calc(2:4), lty=2)
points(poly.calc(-2:6), pch=4)

poly.calc

Calculate Polynomials from Zeros or Values

Description

Calculate either the monic polynomial with specified zeros, or the Lagrange interpolation polynomial through the (x,y) points.

Usage

poly.calc(x, y, tol=sqrt(.Machine$double.eps), lab=dimnames(y)[[2]])

Arguments

x

numeric vector specifying either the zeros of the desired polynomial if this is the only non-missing argument, or the x-values for Lagrange interpolation.

y

numeric vector or matrix specifying the y-values for the Lagrange interpolation polynomial. If y is a matrix, nrow(y) must equal length(x), and each column of y is used separately with x.

tol

An absolute value tolerance, below which coefficients are treated as zero.

lab

If y is a matrix, lab is used as the names vector for the list result.

Details

If y is a matrix, the result is a list of polynomials using each column separately.
If x only is given, repeated zeros are allowed. If x and y are given, repeated values in the x vector must have identical y values associated with them (up to tol), otherwise the first y-value only is used and a warning is issued.

Value

Either a polynomial object, or a list of polynomials, as appropriate. In the latter case the object is of class "polylist".

See Also

polynomial
Examples

poly.calc(rep(1,3))
## -1 + 3*x - 3*x^2 + x^3
poly.calc(0:4, (0:4)^2 + 1)
## 1 + x^2
poly.calc(0:4, cbind(0:4, (0:4)^2 + 1), lab = letters[1:2])
## List of polynomials:
## $a:
## # x
## ## $b:
## # 1 + x^2

\[
\begin{align*}
\text{poly.orth} &\quad \text{Construct Orthogonal Polynomials} \\
\end{align*}
\]

Description

Construct the orthogonal polynomials on a given vector, up to a specified degree.

Usage

\[
poly.orth(x, degree = length(unique(x)) - 1, norm = TRUE)
\]

Arguments

\[
\begin{align*}
x & \quad \text{a numeric vector of abscissae. When evaluated at } x \text{ the polynomials will generate an orthonormal set.} \\
degree & \quad \text{maximum degree required. The default is one fewer than the number of distinct values in } x, \text{ which is maximum possible.} \\
norm & \quad \text{a logical indicating whether the polynomials should be normalized.}
\end{align*}
\]

Value

A list of class "polylist" of objects of class "polynomial" of degree 1, 2, ..., degree.

Examples

\[
x <- rep(1:4, 1:4) \# x with repetitions for weighting
x
## [1] 1 2 2 3 3 4 4 4
\]

polyx <- poly.orth(x, 3) \# calculate orthogonal polynomials
polyx
## List of polynomials:
## [[1]]
## 0.3162278
##
## [[2]]
\]
polylist

Lists of Polynomials

Description

Create and manipulate lists of polynomials.

Usage

polylist(...)  
as.polylist(x)  
is.polylist(x)

Arguments

...  
R objects, polynomials or capable of coercion to polynomials.

x  
an R object.

Details

polylist takes a list of arguments, tries to convert each into a polynomial (see polynomial), and sets the class of the list to "polylist".

as.polylist tries to coerce its arguments to a polylist, and will do so for arguments which are polynomials or lists thereof.

is.polylist tests whether its argument is a polylist.

This class has several useful methods, such as taking derivatives (deriv) and antiderivatives (integral), printing and plotting, subscripting, computing sums and products of the elements, and methods for c, rep, and unique.

Examples

## Calculate orthogonal polynomials
pl <- poly.orth(rep(1:4, 1:4), 3)
pl
plot(pl)
deriv(pl)
integral(pl)
sum(pl)
polynomial

\[
\text{prod}(\text{pl}) \\
\text{unique}(\text{rep}(\text{pl}, 3)\lbrack c(8, 12)\rbrack)
\]

---

**polynomial**

**Polynomials**

**Description**

Construct, coerce to, test for, and print polynomial objects.

**Usage**

```r
polynomial(coef = c(0, 1))
as.polynomial(p)
is.polynomial(p)
```

```r
# S3 method for class 'polynomial'
as.character(x, decreasing = FALSE, ...)
# S3 method for class 'polynomial'
print(x, digits = getOption("digits"), decreasing = FALSE, ...)
```

**Arguments**

- `coef` numeric vector, giving the polynomial coefficients in increasing order.
- `p` an arbitrary R object.
- `x` a polynomial object.
- `decreasing` a logical specifying the order of the terms; in increasing (default) or decreasing powers.
- `digits` the number of significant digits to use for printing.
- `...` potentially further arguments passed to and from other methods.

**Details**

`polynomial` constructs a polynomial from its coefficients, i.e., \( p[1:k] \) specifies the polynomial

\[ p_1 + p_2 x + p_3 x^2 + \ldots + p_k x^{k-1}. \]

Internally, polynomials are simply numeric coefficient vectors of class "polynomial". Several useful methods are available for this class, such as coercion to character (as.character()) and function (as.function.polynomial), extraction of the coefficients (coef()), printing (using as.character), plotting (plot.polynomial), and computing sums and products of arbitrarily many polynomials.

`as.polynomial` tries to coerce its arguments to a polynomial.

`is.polynomial` tests whether its argument is a polynomial (in the sense that it has class "polynomial").
predict.polynomial

Examples

```r
polynomial(1:4)
p <- as.polynomial(c(1,0,3,0))
p
print(p, decreasing = TRUE)
stopifnot(coef(p) == c(1,0,3))
polynomial(c(2,rep(0,10),1))
```

---

predict.polynomial Evaluate a Polynomial

Description

Evaluate a polynomial at a given numeric or polynomial argument.

Usage

```r
## S3 method for class 'polynomial'
predict(object, newdata, ...)
```

Arguments

- `object` A polynomial object to be evaluated.
- `newdata` Argument at which evaluation is requested. May be numeric or itself a polynomial.
- `...` Not used by this method.

Details

This is a method for the generic function `predict`.

The polynomial is evaluated according to the Horner scheme for speed and numerical accuracy.

Value

Evaluated object of the same class as `newdata`.

See Also

`as.function.polynomial`
solve.polynomial  

Zeros of a Polynomial

Description

Find the zeros, if any, of a given polynomial.

Usage

```r
## S3 method for class 'polynomial'
solve(a, b, ...)
```

Arguments

- `a`: A polynomial object for which the zeros are required.
- `b`: A numeric value specifying an additional intercept. If given, the zeros of `a - b` are found.
- `...`: Not used by this method.

Details

This is a method for the generic function `solve`.

The zeros are found as the eigenvalues of the companion matrix, sorted according to their real parts.

Value

A numeric vector, generally complex, of zeros.

See Also

`polyroot`, `poly.calc`, `summary.polynomial`

Examples

```r
p <- polynomial(6:1)
p
## 6 + 5*x + 4*x^2 + 3*x^3 + 2*x^4 + x^5
pz <- solve(p)
pz
## [1] -1.49180+0.00000i -0.80579-1.22291i -0.80579+1.22291i
## [4] 0.55169-1.25331i 0.55169+1.25331i
## To retrieve the original polynomial from the zeros:
poly.calc(pz)
## Warning: imaginary parts discarded in coercion
## 6 + 5*x + 4*x^2 + 3*x^3 + 2*x^4 + x^5
```
**Description**

Summarize a polynomial by describing its “key” points.

**Usage**

```r
## S3 method for class 'polynomial'
summary(object, ...)
```

**Arguments**

- `object`: an object of class "polynomial".
- `...`: Not used by this method.

**Details**

This is a method for the generic function `summary`.

**Value**

A list of class "summary.polynomial" (which has its own print method) containing information on zeros, stationary and inflexion points.

**Examples**

```r
p <- polynomial(6:1)
p
## 6 + 5*x + 4*x^2 + 3*x^3 + 2*x^4 + x^5
pz <- summary(p)
pz
## [1] -1.49180+0.0000i -0.80579-1.22291i -0.80579+1.22291i
## [4]  0.55169-1.25331i  0.55169+1.25331i
## To retrieve the original polynomial from the zeros:
poly.calc(pz)
## Warning: imaginary parts discarded in coercion
## 6 + 5*x + 4*x^2 + 3*x^3 + 2*x^4 + x^5
```
Summary Group Methods for Polynomials

Description

Allows summary group generics to be used on polynomial arguments.

Usage

```r
## S3 method for class 'polynomial'
Summary(..., na.rm = FALSE)
```

Arguments

- `...` R objects, the first supplied of class "polynomial".
- `na.rm` logical: should missing values be removed?

Details

For the `sum` and `prod` functions, the sum and product of the given polynomials, respectively. For the other members of the Summary group, an error is returned.

Note that one could order polynomials by divisibility, and define \texttt{min} and \texttt{max} as the corresponding lattice meet and join, i.e., the greatest common divisor and the least common multiple, respectively. This is currently not provided: instead, functions \texttt{gcd} and \texttt{lcm} should be called directly.

See Also

- `Math.polynomial`, `Ops.polynomial`
Index

*Topic symbolmath
  as.function.polynomial, 2
  change.origin, 3
  deriv.polynomial, 4
  gcd, 5
  integral.polynomial, 5
  lines.polynomial, 6
  Math.polynomial, 7
  monic, 8
  Ops.polynomial, 9
  plot.polynomial, 9
  points.polynomial, 10
  poly.calc, 11
  poly.orth, 12
  polylist, 13
  polynomial, 14
  predict.polynomial, 15
  solve.polynomial, 16
  summary.polynomial, 17
  zsummary.polynomial, 18
  [.polylist (polylist), 13
  as.character.polynomial (polynomial), 14
  as.function, 2
  as.function.polynomial, 2, 14, 15
  as.polylist (polylist), 13
  as.polynomial (polynomial), 14
  c, 13
  c.polylist (polylist), 13
  ceiling, 7
  change.origin, 3, 8
  coef.polynomial (polynomial), 14
  deriv, 4, 13
  deriv.polylist (polylist), 13
  deriv.polynomial, 4, 6
  floor, 7
  GCD, 18
  GCD (gcd), 5
  gcd, 5
  integral, 13
  integral (integral.polynomial), 5
  integral.polylist (polylist), 13
  integral.polynomial, 4, 5
  is.polylist (polylist), 13
  is.polynomial (polynomial), 14
  LCM, 18
  LCM (gcd), 5
  lines, 6, 7, 10, 11
  lines.polynomial, 6, 10, 11
  Math.polynomial, 7, 9, 18
  monic, 8
  Ops.polynomial, 7, 9, 18
  plot, 7, 10, 11
  plot.polylist (polylist), 13
  plot.polynomial, 7, 9, 11, 14
  points, 7, 10, 11
  points.polynomial, 7, 10, 10
  poly.calc, 11, 16
  poly.from.roots (poly.calc), 11
  poly.from.values (poly.calc), 11
  poly.from.zeros (poly.calc), 11
  poly.orth, 12
  polylist, 5, 13
  polynomial, 5, 11, 13, 14
  polyroot, 16
  predict, 15
  predict.polynomial, 2, 15
  print.polylist (polylist), 13
  print.polynomial (polynomial), 14
  print.summary.polynomial (summary.polynomial), 17
  rep, 13
rep.polylist (polylist), 13
round, 7
signif, 7
solve, 16
solve.polynomial, 16
summary, 17
Summary.polynomial, 7, 9
Summary.polynomial
   (zSummary.polynomial), 18
summary.polynomial, 16, 17
trunc, 7
unique, 13
unique.polylist (polylist), 13
zSummary.polynomial, 18