

# Package ‘coga’

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**Title** Convolution of Gamma Distributions

**Version** 1.2.1

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**Description** Evaluation for density and distribution function of convolution of gamma distributions in R. Two related exact methods and one approximate method are implemented with efficient algorithm and C++ code. A quick guide for choosing correct method and usage of this package is given in package vignette. For the detail of methods used in this package, we refer the user to Mathai(1982)<[doi:10.1007/BF02481056](https://doi.org/10.1007/BF02481056)>, Moschopoulos(1984)<[doi:10.1007/BF02481123](https://doi.org/10.1007/BF02481123)>, Barnabani(2017)<[doi:10.1080/03610918.2014.963612](https://doi.org/10.1080/03610918.2014.963612)>, Hu et al.(2020)<[doi:10.1007/s00180-019-00924-9](https://doi.org/10.1007/s00180-019-00924-9)>.

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**License** GPL (>= 3.0)

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**RoxygenNote** 7.2.3

**Suggests** testthat, R.rsp

**Imports** stats, Rcpp, cubature

**LinkingTo** Rcpp, RcppGSL

**VignetteBuilder** R.rsp

**BugReports** <https://github.com/ChaoranHu/coga/issues>

**URL** <https://github.com/ChaoranHu/coga>

**NeedsCompilation** yes

**SystemRequirements** GNU GSL

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coga	<i>coga: Convolution of independent Gamma Distributions</i>
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### Description

Evaluation for density and distribution function of convolution of gamma distributions in R. Two related exact methods and one approximate method are implemented with efficient algorithm and C++ code. A quick guide for choosing correct method and usage of this package is given in package vignette.

### coga functions

dcoga, pcoga, rcoga, dcoga2dim, pcoga2dim, pcoga2dim\_diff\_shape, dcoga\_approx, and pcoga\_approx.

### Author(s)

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dcoga	<i>Convolution of Gamma Distributions (Exact Method).</i>
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### Description

Density, distribution function, and random generation for convolution of gamma distributions. Convolution of independent Gamma random variables is  $Y = X_1 + \dots + X_n$ , where  $X_i, i = 1, \dots, n$ , are independent Gamma distributions with parameters shapes and rates. The exact density function and distribution function can be calculated, according to the formulas from Moschopoulos, Peter G. (1985). *\*\*We mention that dcoga and pcoga are recommended for  $n \geq 3$ .\*\**

### Usage

dcoga(x, shape, rate)

pcoga(x, shape, rate)

rcoga(n, shape, rate)

**Arguments**

x	Quantiles.
shape	Numerical vector of shape parameters for each gamma distributions, all shape parameters should be larger than or equal to 0, with at least one non-zero.
rate	Numerical vector of rate parameters for each gamma distributions, all rate parameters should be larger than 0.
n	Number of sample points.

**Author(s)**

Chaoran Hu

**References**

Moschopoulos, Peter G. "The distribution of the sum of independent gamma random variables." *Annals of the Institute of Statistical Mathematics* 37.1 (1985): 541-544.

**Examples**

```
## Example 1: Correctness check
set.seed(123)
## do grid
y <- rcoga(100000, c(3,4,5), c(2,3,4))
grid <- seq(0, 15, length.out=100)
## calculate pdf and cdf
pdf <- dcoga(grid, shape=c(3,4,5), rate=c(2,3,4))
cdf <- pcoga(grid, shape=c(3,4,5), rate=c(2,3,4))

## plot pdf
plot(density(y), col="blue")
lines(grid, pdf, col="red")

## plot cdf
plot(ecdf(y), col="blue")
lines(grid, cdf, col="red")

## Example 2: Show parameter recycling
## these pairs give us the same results
dcoga(1:5, c(1, 2), c(1, 3, 4, 2, 5))
dcoga(1:5, c(1, 2, 1, 2, 1), c(1, 3, 4, 2, 5))

pcoga(1:5, c(1, 3, 5, 2, 2), c(3, 5))
pcoga(1:5, c(1, 3, 5, 2, 2), c(3, 5, 3, 5, 3))
```

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`dcoga2dim`*Convolution of Two Gamma Distributions (Exact Method).*

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### Description

Density, and distribution function of convolution of *two* gamma distributions. These two functions still give us the exact density and distribution function value, but which are much faster than `dcoga` and `pcoga`. **So, we recommend these two functions for two variables case.** The algorithm of these two functions comes from Mathai, A.M. (1982).

### Usage

```
dcoga2dim(x, shape1, shape2, rate1, rate2)
```

```
pcoga2dim(x, shape1, shape2, rate1, rate2)
```

### Arguments

<code>x</code>	Quantiles.
<code>shape1, shape2</code>	Shape parameters for the first and second gamma distributions, both shape parameters should be larger than or equal to 0, with at least one non-zero.
<code>rate1, rate2</code>	Rate parameters for the first and second gamma distributions, both rate parameters should be larger than 0.

### Author(s)

Chaoran Hu

### References

Mathai, A.M.: Storage capacity of a dam with gamma type inputs. *Ann. Inst. Statist.Math.* 34, 591-597 (1982)

### Examples

```
## Example 1: Correctness check
set.seed(123)
## do grid
y <- rcoga(100000, c(3,4), c(2,3))
grid <- seq(0, 15, length.out=100)
## calculate pdf and cdf
pdf <- dcoga2dim(grid, 3, 4, 2, 3)
cdf <- pcoga2dim(grid, 3, 4, 2, 3)

## plot pdf
plot(density(y), col="blue")
lines(grid, pdf, col="red")
```

```
## plot cdf
plot(ecdf(y), col="blue")
lines(grid, cdf, col="red")

## Example 2: Comparison with `dcoga` and `pcoga`
## these pairs give us the same results
dcoga(1:5, c(1, 2), c(3, 4))
dcoga2dim(1:5, 1, 2, 3, 4)

pcoga(1:5, c(1, 3), c(3, 5))
pcoga2dim(1:5, 1, 3, 3, 5)
```

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dcoga\_approx

*Convolution of Gamma distribuitons (Approximation Method)*

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## Description

Density and distribution function of convolution of gamma distributions are calculated based on approximation method from Barnabani(2017), which gives us the approximate result and faster evaluation than dcoga and pcoga during three or more variables case. **\*\*So, we recommend these functions for three or more varibales case with approximate result.\*\***

## Usage

```
dcoga_approx(x, shape, rate)
```

```
pcoga_approx(x, shape, rate)
```

## Arguments

x	Quantiles.
shape	Numerical vector of shape parameters for each gamma distributions, all shape parameters should be larger than or equal to 0, with at least three non-zero.
rate	Numerical vector of rate parameters for each gamma distributions, all rate parameters should be larger than 0.

## Author(s)

Chaoran Hu

## References

Barnabani, M. (2017). An approximation to the convolution of gamma distributions. *Communications in Statistics - Simulation and Computation* 46(1), 331-343.

**Examples**

```

## Example 1: Correctness check
set.seed(123)
## do grid
y <- rcoga(100000, c(3,4,5), c(2,3,4))
grid <- seq(0, 15, length.out=100)
## calculate pdf and cdf
pdf <- dcoga_approx(grid, shape=c(3,4,5), rate=c(2,3,4))
cdf <- pcoga_approx(grid, shape=c(3,4,5), rate=c(2,3,4))

## plot pdf
plot(density(y), col="blue")
lines(grid, pdf, col="red")

## plot cdf
plot(ecdf(y), col="blue")
lines(grid, cdf, col="red")

## Example 2: Show parameter recycling
## these pairs give us the same results
dcoga_approx(1:5, c(1, 2), c(1, 3, 4, 2, 5))
dcoga_approx(1:5, c(1, 2, 1, 2, 1), c(1, 3, 4, 2, 5))

pcoga_approx(1:5, c(1, 3, 5, 2, 2), c(3, 5))
pcoga_approx(1:5, c(1, 3, 5, 2, 2), c(3, 5, 3, 5, 3))

```

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pcoga2dim\_diff\_shape    *Recurrence Identity of Shape Parameter for pcoga2dim*

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**Description**

The difference of distribution functions of convolution of two gamma distributions between consecutive neighbors of shape parameter. This function evaluate the value of 'pcoga2dim(x, shape1, shape2, rate1, rate2) - pcoga2dim(x, shape1 + 1, shape2, rate1, rate2)' with higher efficiency (this function is much more faster than call pcoga2dim two-times).

**Usage**

```
pcoga2dim_diff_shape(x, shape1, shape2, rate1, rate2)
```

**Arguments**

x	Quantiles.
shape1, shape2	Shape parameters of the first and second gamma distributions, both shape parameters should be larger than or equal to 0.
rate1, rate2	Rate parameters of the first and second gamma distributions, both rate parameters should be larger than 0.

**Author(s)**

Chaoran Hu

**Examples**

```
## these pairs give us the same results
pcoga2dim_diff_shape(3,2,4,5,4)
pcoga2dim(3,2,4,5,4) - pcoga2dim(3,3,4,5,4)
```

```
pcoga2dim_diff_shape(3,0,4,3,5)
pgamma(3,4,5) - pcoga2dim(3,1,4,3,5)
```

```
pcoga2dim_diff_shape(3,6,0,5,4)
pgamma(3,6,5) - pgamma(3,7,5)
```

```
pcoga2dim_diff_shape(3,0,0,4,5)
1 - pgamma(3,1,4)
```

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