# Package ‘contfrac’

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**Title**  Continued Fractions  
**Version**  1.1-12  
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**Description**  Various utilities for evaluating continued fractions.  
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## Description

Approximates a real number in continued fraction form using a standard simple algorithm

## Usage

```
as_cf(x, n = 10)
```
Arguments

- **x**: real number to be approximated in continued fraction form
- **n**: Number of partial denominators to evaluate; see Notes

**Note**

Has difficulties with rational values as expected

**Author(s)**
Robin K. S. Hankin

**See Also**

- CF, convergents

**Examples**

```r
phi <- (sqrt(5)+1)/2
as_cf(phi,50)  # loses it after about 38 iterations ... not bad ...

as_cf(pi)  # looks about right
as_cf(exp(1),20)

f <- function(x){CF(as_cf(x,30),TRUE) - x}
x <- runif(40)
plot(sapply(x,f))
```

---

**Description**

Returns continued fraction convergent using the modified Lenz’s algorithm; function CF() deals with continued fractions and GCF() deals with generalized continued fractions.

**Usage**

- `CF(a, finite = FALSE, tol=0)`
- `GCF(a,b, b0=0, finite = FALSE, tol=0)`
Arguments

a, b
In function CF(), the elements of a are the partial denominators; in GCF() the elements of a are the partial numerators and the elements of b the partial denominators

finite
Boolean, with default FALSE meaning to iterate Lenz’s algorithm until convergence (a warning is given if the sequence has not converged); and TRUE meaning to evaluate the finite continued fraction

b0
In function GCF(), floor of the continued fraction

tol
tolerance, with default 0 silently replaced with .Machine$double.eps

Details

Function CF() treats the first element of its argument as the integer part of the convergent.
Function CF() is a wrapper for GCF(); it includes special dispensation for infinite values (in which case the value of the appropriate finite CF is returned).
The implementation is in C; the real and complex cases are treated separately in the interests of efficiency.
The algorithm terminates when the convergence criterion is achieved irrespective of the value of finite.

Author(s)

Robin K. S. Hankin

References


See Also

convergents

Examples

phi <- (sqrt(5)+1)/2
phi_cf <- CF(rep(1,100))  # phi = [1;1,1,1,1,...]
phi - phi_cf  # should be small

# The tan function:
"tan_cf" <- function(z,n=20){
  GCF(c(z, rep(-z^2,n-1)), seq(from=1,by=2, len=n))
}
z <- 1+1i
\[ \tan(z) - \tan_{cf}(z) \quad \# \text{should be small} \]

# approximate real numbers with continued fraction:
\[ \text{as}_{-}\text{cf}(\pi) \]

\[ \text{as}_{-}\text{cf}(\exp(1), 25) \quad \# \text{OK up to element 21 (which should be 14)} \]

# Some convergents of \( \pi \):
jj <- \text{convergents}(c(3, 7, 15, 1, 292))
jj$A / jj$B - \pi

# An identity of Euler's:
jj <- \text{GCF}(a = \text{seq}(from = 2, by = 2, len = 30), b = \text{seq}(from = 3, by = 2, len = 30), b0 = 1)
jj - 1 / (\exp(0.5) - 1) \quad \# \text{should be small}

---

### convergents

**Partial convergents of continued fractions**

**Description**

Partial convergents of continued fractions or generalized continued fractions

**Usage**

\[
\text{convergents}(a) \\
\text{gconvergents}(a, b, b0 = 0)
\]

**Arguments**

- **a, b**
  - In function `convergents()`, the elements of `a` are the partial denominators (the first element of `a` is the integer part of the continued fraction). In `gconvergents()` the elements of `a` are the partial numerators and the elements of `b` the partial denominators
  - `b0` The floor of the fraction

**Details**

Function `convergents()` returns partial convergents of the continued fraction

\[
a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4 + \ddots}}}}
\]

where \( a = a_0, a_1, a_2, \ldots \) (note the off-by-one issue).
convergents

Function gconvergents() returns partial convergents of the continued fraction

\[ b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \ddots}}} \]

where \( a = a_1, a_2, \ldots \)

Value

Returns a list of two elements, A for the numerators and B for the denominators

Note

This classical algorithm generates very large partial numerators and denominators. To evaluate limits, use functions CF() or GCF().

Author(s)

Robin K. S. Hankin

References


See Also

CF

Examples

# Successive approximations to pi:

jj <- convergents(c(3,7,15,1,292))
jj$A/jj$B - pi  # should get smaller

convergents(rep(1,10))
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